

# Introduction

The *Massachusetts Curriculum Framework for Mathematics* builds on the *Common Core State Standards for Mathematics*. The standards in this *Framework* are the culmination of an extended, broad-based effort to fulfill the charge issued by the states to create the next generation of pre-kindergarten through grade 12 standards in order to help ensure that all students are college and career ready in mathematics no later than the end of high school.

In 2008 the Massachusetts Department of Elementary and Secondary Education convened a team of educators to revise the existing *Massachusetts Mathematics Curriculum Framework* and, when the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practice (NGA) began a multi-state standards development initiative in 2009, the two efforts merged. The *Common Core State Standards for Mathematics* were adopted by the Massachusetts Board of Elementary and Secondary Education on July 21, 2010.

In their design and content, refined through successive drafts and numerous rounds of feedback, the standards in this document represent a synthesis of the best elements of standards-related work to date and an important advance over that previous work. As specified by CCSSO and NGA, the standards are (1) research- and evidence-based, (2) aligned with college and work expectations, (3) rigorous, and (4) internationally benchmarked. A particular standard was included in the document only when the best available evidence indicated that its mastery was essential for college and career readiness in a twenty-first-century, globally competitive society. The standards are intended to be a living work: as new and better evidence emerges, the standards will be revised accordingly.

## Unique Massachusetts Standards and Features

The *Massachusetts Curriculum Framework for Mathematics* incorporates the *Common Core State Standards* and a select number of additional standards unique to Massachusetts (coded with an initial “MA” preceding the standard number), as well as additional features unique to Massachusetts that add further clarity and coherence to the *Common Core* standards. These unique Massachusetts elements include standards for pre-kindergartners; Guiding Principles for mathematics programs; expansions of the *Common Core*’s glossary and bibliography; and an adaptation of the high school model courses from the *Common Core State Standards for Mathematics Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards*.

Staff at the Massachusetts Department of Elementary and Secondary Education worked closely with the Common Core writing team to ensure that the standards are comprehensive and organized in ways to make them useful for teachers. The pre-kindergarten standards were adopted by the Massachusetts Board of Early Education and Care on December 14, 2010.

## Toward Greater Focus and Coherence

For over a decade, research studies conducted on mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” The standards in this *Framework* are a substantial answer to that challenge and aim for clarity and specificity.

William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a

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means for connecting the particulars (such as an understanding of the rational number system and its properties). (emphasis added)

The development of these standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skills, and understanding develop over time.

The standards do not dictate curriculum or teaching methods. In fact, standards from different domains and clusters are sometimes closely related. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

## Highlights of the 2011 *Massachusetts Curriculum Framework for Mathematics*

- Guiding Principles for Mathematics Programs, revised from the past Massachusetts *Mathematics Framework*, now show a strong connection to the Standards for Mathematical Practice.
- New Standards for Mathematical Practice describe mathematically proficient students, and should be a part of the instructional program along with the content standards.
- In contrast to earlier Massachusetts mathematics content standards, which were grouped by grade spans, the pre-kindergarten to grade 8 content standards in this document are written for individual grades.
  - The introduction at each grade level articulates a small number of critical mathematical areas that should be the focus for that grade.
  - A stronger middle school progression includes new and rigorous grade 8 standards that encompass some standards covered in the 2000 Algebra I course.
  - These pre-kindergarten through grade 8 mathematics standards present a coherent progression and a strong foundation that will prepare students for the 2011 Model Algebra I course. Students will need to progress through the grade 8 mathematics standards in order to be prepared for the 2011 Model Algebra I course.
- At the high school level, standards are grouped into six conceptual categories, each of which is further divided into domain groupings.
  - In response to many educators' requests to provide models for how standards can be configured into high school courses, this Massachusetts *Framework* also presents eight model courses for high school standards, featuring two primary pathways:
    - Traditional Pathway (Algebra I, Geometry, Algebra II);
    - Integrated Pathway (Mathematics I, Mathematics II, Mathematics III); and
    - Also included are two additional advanced model courses (Precalculus, Advanced Quantitative Reasoning).

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- The following supplementary resources are included in this *Framework*.
  - Application of Common Core State Standards for English Language Learners (from the *Common Core State Standards*);
  - Application of Common Core State Standards for Students with Disabilities (from the *Common Core State Standards*);
  - An updated Glossary of Mathematical Terms; and
  - Sample of Works Consulted.

## Document Organization

Six **Guiding Principles for Mathematical Programs in Massachusetts** follow this introductory section. The Guiding Principles are philosophical statements that underlie the standards and resources in this *Curriculum Framework*.

Following the Guiding Principles are the eight **Standards for Mathematical Practice** that describe the varieties of expertise that all mathematics educators at all levels should seek to develop in their students.

The **Standards for Mathematical Content** (learning standards) are next in the document, and are presented in three sections:

- Pre-kindergarten through grade 8 content standards are presented by *grade level*;
- High school content standards are presented by *conceptual category*; and
- High school content standards are also presented through *model high school courses*—six model courses outlined in two pathways (Traditional and Integrated) and two model advanced courses.

The **supplementary resources** that follow the learning standards address how to apply the standards for English language learners and students with disabilities. The glossary and list of references from the *Common Core State Standards* are also included and expanded with Massachusetts additions.

# **The Standards for Mathematical Practice**



# The Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on the following two sets of important “processes and proficiencies,” each of which has longstanding importance in mathematics education:

- The NCTM process standards
  - problem solving
  - reasoning and proof
  - communication
  - representation
  - connections
- The strands of mathematical proficiency specified in the National Research Council’s report “Adding It Up”
  - adaptive reasoning
  - strategic competence
  - conceptual understanding (comprehension of mathematical concepts, operations, and relations)
  - procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately)
  - productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy)

## The Standards for Mathematical Practice

### 1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### 3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is

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a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## 7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

# The Standards for Mathematical Practice

## 8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.



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In grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

- (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
- (2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $\frac{15}{9} = \frac{5}{3}$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
- (3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

# Grade 4

## Overview

### Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

### Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

### Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

### Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: Understand concepts of angle and measure angles.

### Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

### STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for an express regularity in repeated reasoning.

**Content Standards****Operations and Algebraic Thinking****4.OA****Use the four operations with whole numbers to solve problems.**

1. Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.<sup>24</sup>
3. Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

**Gain familiarity with factors and multiples.**

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

**Generate and analyze patterns.**

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

**Number and Operations in Base Ten<sup>25</sup>****4.NBT****Generalize place value understanding for multi-digit whole numbers.**

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

**Use place value understanding and properties of operations to perform multi-digit arithmetic.**

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. MA.5a. Know multiplication facts and related division facts through  $12 \times 12$ .
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

<sup>24</sup> See Glossary, Table 2.<sup>25</sup> Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

**Extend understanding of fraction equivalence and ordering.**

1. Explain why a fraction  $\frac{a}{b}$  is equivalent to a fraction  $\frac{n \times a}{n \times b}$  by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

3. Understand a fraction  $\frac{a}{b}$  with  $a > 1$  as a sum of fractions  $\frac{1}{b}$ .
  - a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
  - b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ ;  $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$ ;  $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$ .
  - c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
  - d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
  - a. Understand a fraction  $\frac{a}{b}$  as a multiple of  $\frac{1}{b}$ . *For example, use a visual fraction model to represent  $\frac{5}{4}$  as the product  $5 \times (\frac{1}{4})$ , recording the conclusion by the equation  $\frac{5}{4} = 5 \times (\frac{1}{4})$ .*
  - b. Understand a multiple of  $\frac{a}{b}$  as a multiple of  $\frac{1}{b}$ , and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express  $3 \times (\frac{2}{5})$  as  $6 \times (\frac{1}{5})$ , recognizing this product as  $\frac{6}{5}$ . (In general,  $n \times (\frac{a}{b}) = (n \times a)/b$ .)*
  - c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $\frac{3}{8}$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

**Understand decimal notation for fractions, and compare decimal fractions.**

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.<sup>27</sup> *For example, express  $\frac{3}{10}$  as  $\frac{30}{100}$ , and add  $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$ .*
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as  $\frac{62}{100}$ ; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model.

<sup>26</sup> Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

<sup>27</sup> Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

**Measurement and Data****4.MD****Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**

1. Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

**Represent and interpret data.**

4. Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

**Geometric measurement: Understand concepts of angle and measure angles.**

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
  - a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $\frac{1}{360}$  of a circle is called a “one-degree angle,” and can be used to measure angles.
  - b. An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees.
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

**Geometry****4.G****Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.